Impedance matching by the combined effects of the outer and middle ear

Mead C. Killion[a] and Peter Dallos

Auditory Research Laboratories, Northwestern University, Evanston, Illinois 60201

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When the impedance mismatch between air and the oval window is computed by considering the latter as a small piston in a baffle, and taking the appropriate source impedance into account, it can be shown that the mismatch is frequency dependent and may be greater than 50 dB at 100 Hz. The middle ear transformer by itself is inadequate to compensate for the loss of transmission due to the mismatch. Only at the frequency of the outer ear resonance does the *combined* action of the outer ear and the middle ear produce an estimated gain that roughly compensates for the loss of energy at the interface. Detailed computations are shown for cats, and some results applicable to the human ear are also included.

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As a first approximation to the impedance mismatch between the cochlear fluid and air, Wever and Lawrence (1954) simply assumed a situation analogous to transmission from one acoustic medium into another at an extended interface. Thus they took the ratio of the specific acoustic resistances of the two media: 41.5 dyn s cm^{-3} for air and 161 000 dyn s cm^{-3} for cochlear fluid (assumed similar to sea water). They estimated that the resulting impedance mismatch of 3880:1 would result in a 30-dB loss of energy were it not for the amplification due to the middle ear. The middle ear transformer ratio was estimated as 18.3:1, yielding a reduced loss of only 5 dB. This large-area approximation was subsequently popularized by several authors, including one of us (Dallos, 1973). The failure of this approach has recently been clarified by Schubert (1978).

In order to compute the actual impedance mismatch, one needs to know the acoustic input impedance of the oval window and the source impedance of the medium from where sound impinges upon the window. If one would again take the specific impedance of the cochlear fluid (161 000 cgs units) and use the dimensions of the oval window, approximately 0.032 cm^2 (Békésy and Rosenblith, 1951), an acoustic input impedance could be computed. A value of 5×10^6 dyn s cm^{-5} results which is within the range of available measurements (Tonndorf *et al.*, 1966; Zwislocki, 1975). While the resulting value is approximately correct, the underlying computation is not. The dimensions of the fluid-filled cochlear chamber are so small and the effect of the round window pressure release is so important, that the simple computation is not permissible. Similarly, a computation of the source impedance obtained by dividing the specific resistance of air with the area of the oval window (yielding 1 300 cgs Ω) is even less appropriate because the dimensions of the oval window are small compared to the wavelength at all audible frequencies.

In the subsequent treatment of the impedance matching problem, we are relying on experimentally determined values for cochlear input impedance as summarized by Zwislocki (1975). A theoretical treatment of the source impedance is possible. Assuming that a plane wave (of airborne sound) is directly incident on the oval window, it is possible to calculate quite accurately the source impedance which should be seen by the window. Raleigh (1894) provided an early derivation of the correct result for such a problem, and Bauer (1966) has published an elegant approximation accurate to within 10%–20%. Thus, we proceed by assuming that the oval window is equivalent to a small piston mounted in an infinite baffle (the head). The source impedance seen by the piston is equivalent to its radiation impedance, which is given by, among others, Olson (1947) and Kinsler and Frey (1962).

Let us assume for simplicity that the inner ear is immediately below the surface of the skull and that it communicates with the environment via a small window, flush with the surface. Assume further that the surface area of this opening is the same as that of the stapes footplate, 0.031 cm^2 (Békésy and Rosenblith, 1951). The radiation impedance of a small piston may be approximated by the first terms in the real and imaginary Bessel series expansions, a truncation yielding less than 1% error below 10 kHz in the present case because the dimensions of the piston are much smaller than the wavelength. The approximation leads to the expression

$$Z_R = (2\pi\rho/c)f^2 + j(16\rho/3\pi a)f, \tag{1}$$

where Z_R is the acoustic radiation impedance, with dimensions of dyn s cm^{-5}, or cgs acoustic ohms. In this expression ρ is the density of air (0.0012 g cm^{-3}), c is the velocity of sound in air (34 300 cm s^{-1}), a is the equivalent radius of the piston, here 0.1 cm, and f is frequency. The second term is dominant for all frequencies of interest, thus,

$$Z_R \approx j(16\rho/3\pi a)f. \tag{2}$$

Substitution of the appropriate constants yields:

$$Z_R = 2 \times 10^{-7} f^2 + j\,2 \times 10^{-2} f \approx j\,2 \times 10^{-2} f. \tag{3}$$

It is significant to note that whereas the traditional large-area treatment implies a resistive source, the actual source impedance is dominated by a frequency dependent mass-reactance term.

A similar result obtains if one utilizes the elegant

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[a] Also with Industrial Research Products, Inc., a Knowles Company, 321 N. Bond St., Elk Grove Village, IL 60007.

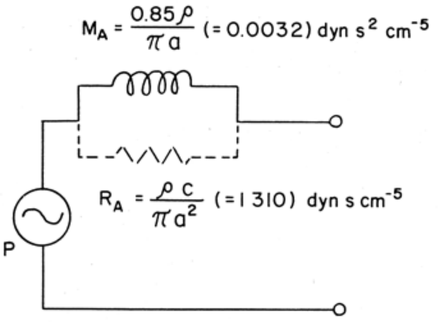

FIG. 1. Equivalent circuit representing the source impedance seen by a small piston [after Bauer (1967)]. The computed values, shown in parentheses, apply to a piston equivalent in area to the human stapes. For most frequencies of interest the resistive element (shown with interrupted lines) is negligible.

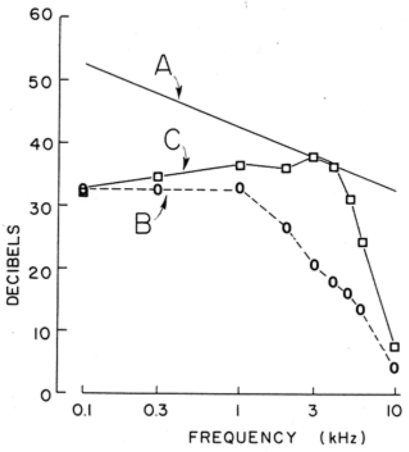

FIG. 2. Graphical representation of some data shown in Table I. A: Computed impedance mismatch beteen air and the cat's oval window. B: Gain due to the amplification by the middle ear alone. C: Gain due to the combined effects of middle ear and external ear. The difference between plots A and either B or C is an indication of the residual impedance mismatch that exists in spite of amplification by the middle ear (B) or by the external plus middle ear (C).

lumped-parameter approximation of the radiation impedance that was derived by Bauer (1966). This circuit, shown in Fig. 1, is a parallel combination of acoustic resistance and mass elements.[1] The approximation is valid within 20% throughout the frequency range. The acoustic mass shown in Fig. 1 has a reactance of approximately 2, 20, and 200 cgs Ω at 100, 1000, and 10 000 Hz, respectively, while the resistive component is approximately 1300 cgs Ω. Again we see that the resistive component is negligible in the parallel combination and that the source impedance is nearly a pure reactance.

A transformer designed to match the acoustic input impedance of the cochlea (Z_C) to the radiation impedance (Z_R) would need to provide a transformation of pressures: $N = (Z_C/Z_R)^{1/2}$. The acoustic input impedance of the cochlea has been estimated by several investigators. A survey of these results has recently been made by Zwislocki (1975). A value that appears appropriate is $Z_C = 0.35 \times 10^6$ cgs Ω. Since the source impedance is reactive, so is the transformed impedance. Thus, the pressure developed across the (resistive) cochlear input impedance is $P_C = P_d N/(2)^{1/2}$, where P_d is the input pressure. We are interested in the necessary pressure transformation (P_C/P_d) by a middle ear that would provide perfect impedance matching. Substitution yields:

$$P_C/P_d \approx 3 \times 10^3/(f)^{1/2}. \qquad (4)$$

Thus, for example, at 100 Hz the required pressure transformation is 300-fold, or 50 dB, considerably in excess of the 30 dB predicted by the large-area approximation.

It is of interest to consider how well the middle ear compensates for the potential impedance mismatch, and loss, at various frequencies. We are showing some computations for cats, inasmuch as their middle ear transfer function is relatively well known. Before scrutinizing the effectiveness of the middle ear, however, the required or ideal transformation is computed. The cochlear input impedance of the cat is estimated as $Z_C = 1.1 \times 10^6$ dyn s cm^{-5} (Zwislocki, 1975). The area of the stapes footplate is 1.26×10^{-2} cm^2 (Guinan and Peake, 1967), thus, $a = 0.064$ cm. Substitution into (2) results in an expression for the source impedance:

$$Z_R \approx j\ 3.1 \times 10^{-2} f. \qquad (5)$$

Thus, the required transformation to overcome the impedance mismatch is

$$P_C/P_d \approx 4.2 \times 10^3/(f)^{1/2}. \qquad (6)$$

The computed values are the desired pressure transformation if a perfect match between surrounding air and the piston (oval window) is to be achieved. These values are given in the first row of Table I for several frequencies; they are also plotted in Fig. 2. It is notable that the necessary transformation of pressure is quite large at low frequencies (52.5 dB at 100 Hz) and it decreases at a rate of 10 dB/decade. Only at high frequencies do the numbers approach the 30-dB impedance mismatch predicted by the simple large-area approximation.

In cats the amplification due to the middle ear at low frequencies can be estimated as the product of pressure gain (21.4-fold) and force gain (twofold) on the basis of Guinan and Peake's 1967 data. Thus, a total dc gain of 42.8, or 32.6 dB, can be expected from the middle-ear transformer. This gain clearly falls short of the desired amplification.[2] At frequencies other than the very low ones, the losses inherent in middle ear mechanics need to be taken into account as well. In Table I the second row shows the actual middle ear gain, i.e., the dc gain of 32.6 dB minus the frequency-dependent middle-ear transfer function magnitude that can be estimated from the data of Guinan and Peake (1967). These numbers are also plotted in Fig. 2. In row 3 of Table I, the difference between the figures shown in rows one and two are given. These numbers reflect the deficit from perfect match between inner ear and air if *only* the middle ear is supplying a gain. It is apparent that the middle ear falls quite short in providing complete compensation for the loss due to disparate impedances at the oval window. Data are

TABLE I. Computational results showing the theoretical frequency dependence of the impedance mismatch between air and the cat's oval window (top row, line A in Fig. 2). The second row provides the gain due to the cat's middle ear (derived from the data of Guinan and Peake, 1967). These data are given as graph B in Fig. 2. In the third row the deficit from perfect match is shown if impedance transformation is affected by the middle ear alone. In row four the gain due to external ear resonances and head baffle effects are shown (from Wiener *et al.*, 1966). In row five the total gain due to the combined effects of external and middle ears is given. (Shown as graph C in Fig. 2.) Finally, in the bottom row the deficit from perfect match is shown if *both* middle ear and external ear are effective in boosting the pressure at the oval window. All entries in the table are in dB.

	Frequency (kHz)								
	0.1	0.3	1.0	2.0	3.0	4.0	5.0	6.0	10.0
Pressure loss due to impedance mismatch	52.5	47.8	42.5	39.5	37.8	36.5	35.5	34.7	32.5
Middle ear gain: 32.6 dB minus frequency dependent middle ear loss	32.6	32.6	32.6	26.6	20.2	17.5	16.1	13.9	3.6
Deficit from perfect match if only middle ear is effective	19.9	15.2	9.9	12.9	17.6	19.0	19.4	20.8	28.9
External ear gain	0	2	4	9	18	19	15	11	4
Middle ear plus external ear gain	32.6	34.6	36.6	35.6	38.2	36.5	31.1	24.9	7.6
Deficit from perfect match if both middle ear and external ear contribute	19.9	13.2	5.9	3.9	−0.4	0.0	4.4	9.8	24.9

available on the sound-pressure transformation between free field and the eardrum in cats (Wiener *et al.*, 1966). The appropriate numbers for 0° incidence are included in row 4 of Table I. If aside from the middle ear effects, the amplification due to external ear resonance is also taken into account, then the total gain shown in row 5 of Table I and also in Fig. 2 results. One can recompute the discrepancy between perfect match and the actual sound transformation. These figures comprise row 6 of Table I. It is interesting that the *combined* amplification due to the middle and external ear yields essentially perfect impedance match for cats between 3–4 kHz.

In human beings the external ear resonance is at 2.7 kHz (Shaw, 1974) where it provides nearly 15 dB of gain (eardrum pressure compared with diffuse-field pressure for random-incidence sound). Although the low-frequency pressure gain of the human middle ear has been estimated at 25 dB (Békésy, 1960; Zwislocki, 1975), certainly at 2.7 kHz a smaller number must be used due to the filter characteristics of the middle ear. It is known, for example, that at that frequency the eardrum motion is no longer completely coherent (Tonndorf and Khanna, 1972). An impedance match equivalent to an approximately 35-dB pressure gain is required at 2.7 kHz to compensate for losses in the transmission from the surrounding medium into the cochlea [see Eq. (4).] Thus, the *combined* action of the human external and middle ears probably results in a reasonably good impedance match between the complete ear and its environment in the vicinity of the ear's best frequency.

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[1] In order to account for diffraction, Bauer's complete equivalent-circuit approximation contains an additional source of pressure $2P$. At frequencies where the piston is small compared to the wavelength, as in the present case, the simplification shown in Fig. 1 is adequate.

[2] A similar estimate of the low-frequency middle ear transformer ratio can be obtained from the Khanna and Tonndorf (1972) data on the volume displacement of the eardrum and the Guinan and Peake (1967) data on the volume displacement of the stapes footplate, since both sets of data are given for a known pressure at the eardrum. Per unit pressure, the former is approximately 2.5×10^{-7} cm^5/dyn, while the latter is approximately 5×10^{-9} cm^5/dyn. Assuming there is no lost motion, the ratio of the two volume displacements gives an estimated transformer ratio of 50, or a gain of 34 dB. Even if one accepts the overall transformer ratio of 87 estimated by Khanna and Tonndorf assuming a lossless middle ear, the middle ear gain is still inadequate.

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