Measurement of Individual Loudness Functions by Trisection of Loudness Ranges

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Objectives: Loudness-balance measurements with monaurally impaired subjects have shown that the shape of the loudness versus sound-pressure curve among hearing-impaired persons varies significantly. But the effectiveness of adjusting the compression characteristics of wide-dynamic-range compression hearing aids—the compression ratios, the variation of compression ratio with level, and the threshold of compression—to restore normal loudness growth for the individual patient has never been properly tested; individual loudness measurements have been too uncertain to permit meaningful individual adjustments. Recent investigators have reported standard deviations of such measurements in normal-hearing subjects of 6.4 dB and 7.8 dB. This investigation describes a method of measuring loudness function with a standard deviation in normal-hearing subjects of the order of 1 dB, both significantly lower than that of previous methods and sufficiently accurate for individual-subject adjustments.

Design: Each of nine normal-hearing subjects—seven of them inexperienced and one a 9-year-old—was asked to make three successive loudness trisections within an amplitude range of 40 to 80 dB SPL, providing six points from which to plot a loudness-function curve between these limits. The individual and average curves were validated as accurate loudness functions by comparing them to the curve defined by the equation of loudness versus amplitude in current Standards. In a second validation experiment, the loudness functions of masked ears measured by trisection were compared to the loudness function of those ears measured by loudness balance between masked and unmasked ears.

Results: The difference between a loudness function based on the average of subject trisections and the loudness function defined by the ANSI Standard loudness equation was −1.92 dB at the lowest trisection level and +0.05 dB at the highest level. The standard deviations of subject responses were 1.63 dB for the lowest trisection level and 0.88 dB for the highest level, with an average of 1.1 dB. The across-subject standard deviation of the test-retest differences for three subjects was less than 1.7 dB for the first three lower level responses and less than 0.8 dB for the remaining three responses.

Conclusions: A trisection procedure for measuring loudness function showed validity and significantly less variation than previous loudness-measurement procedures. Such a procedure, once it has been validated for hearing-impaired subjects, makes it possible to test hearing aid design and fitting strategies that are based on individual-patient loudness functions.

(Ear & Hearing 2008;29:693–703)

INTRODUCTION

This study describes and tests a procedure for measuring in detail the shape of the loudness function (see footnote 1) of individual subjects, using loudness-interval judgments. The procedure may have more than academic interest: compression amplification in hearing aids is designed to compensate for the abnormal loudness function created by recruitment, and knowing the shape of a patient’s abnormal loudness vs. sound-pressure curve could prove useful in designing and adjusting accurate compensatory electronic circuits.

The procedure was tested on normal-hearing subjects, in quiet and with masking. We would not expect the trisection procedure to work differently for hearing impaired than for normal-hearing subjects; binaural loudness-balance, threshold, and suprathreshold equal-loudness measurements (Villchur, 1973) work equally well for either type of subject. But a procedure that will be used on hearing-impaired subjects must be tested on such subjects, which remains to be done.

The measurements of early investigators of recruitment showed that the shape of the loudness vs. sound-pressure curve among hearing-impaired subjects with recruitment varied significantly. Harris et al., (1952) measured recruitment in monaurally hearing-impaired subjects, who adjusted test-signal levels for the same loudness in their impaired and normal ears. Harris et al., reported that the subjects’ recruitment curves (which plot the variation of recruitment with level) were of four main types: asympotic to the normal loudness function, straight-line, delayed, and delayed plus asymptotic. Recruitment curves representative of three of these types, two measured by Harris et al., and the third (the asymp-

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1The curve of loudness versus physical sound level.
Fig. 1. Three types of recruitment curves defined by Harris et al. as asymptotic, straight-line, and delayed. The hearing loss at a particular level is the horizontal or vertical distance between the recruitment curve and the diagonal normal-hearing reference. From Harris et al. (1952).

Garner (1954) proposed a normal loudness function that he measured by fractionation (subjects estimate the loudness ratio between two tones of different amplitude) and equestation (subjects estimate equal loudness intervals, but unlike the procedure of this study the tones are presented one at a time rather than as part of a series). His loudness equation predicted that loudness was doubled with an increase of 25 phons rather than the 10 phons of current standards (ANSI Standard S3.20-1995, ISO R131—1959E). He substituted “lambda” loudness units for sones, and reported an average standard deviation of 0.13 in log lambda units over the range of 50 to 110 phons. This is equivalent to a standard deviation of 9.9 phons, based on Garner’s stated standard deviation and his loudness equation.

Allen et al. (1990) developed a procedure for measuring individual loudness functions which they called the Loudness Growth in 1/2-Octave Bands test, in which the subject judged the relative loudness of randomly presented signals of different amplitudes in different frequency bands. The Loudness Growth in Octave Bands procedure was the first to use measured loudness functions to calculate compression ratios for hearing aids. Allen et al. reported a standard deviation of the responses of 15 normal-hearing subjects of 7.8 db, and a within-subject standard deviation of 2.9 db for six subjects, part of which Allen et al. attributed to variation in the transducer coupling to the subjects’ ears.

Hellman and Meiselman (1993) measured the loudness functions of both normal and hearing-impaired listeners, using absolute magnitude estimation, absolute magnitude production, and cross-modality matching. They reported the intersubject standard deviation of the responses of 83 normal-hearing subjects in terms of the variation in slope of the loudness function plotted as a straight line. The standard deviation was 0.13 from a central value of 0.6, which may be translated to 6.4 db in terms of SPL subject responses (see footnote 2). Part of this variation was probably caused by the MX-41/AR earphone cushions that were used.

In the present experiment, individual loudness functions are plotted from the subject’s division of loudness ranges. Stevens and Davis (1983), in discussing such divisions, wrote: “... bisecting a loudness interval, one can aim either at setting the middle tone halfway between the other two or at an adjustment such that the ratio of the middle tone to the lowest tone equals the ratio of the highest tone to the middle tone. In other words, one can aim either at the arithmetic or at the geometric mean. Different results are obtained by observers having these two attitudes.” Stevens and Davis provided an example of the difference between the two types of bisection: the arithmetic mean of the loudness interval between 5 and 20 sones (see footnote 3) is 12.5 sones, 7.5 sones to each part of the bisected interval; the geometric mean is 10 sones (5 x 2), 5 sones to the first part of the bisected interval and 10 sones to the second part.

The normal loudness function becomes a straight line with a slope of 0.6 (the same value as the exponent of the Stevens (1955) loudness equation; see footnote 6) if the sone scale is spaced geometrically and its units are then converted to dB ratios re 1 sone. The standard deviation of subject responses in terms of loudness-function slope may be converted to SPL-response values by calculating, at a particular input stimulus, the difference between the SPL response that creates the average slope and the SPL response that creates the variant slope. Between 40 dB and 60 dB SPL input the normal 0.6 slope may be read as 12 dB/20 dB, which is the dB ratio of 4 sones re 1 sone divided by the 20-dB range. If the slope is increased to 0.73, the relation between the sone dB ratio and the SPL range becomes 14.92. 14.6 dB above 1 sone is 5.37 sones, corresponding to 64.3 dB SPL and 64.3 to 60 = 4.3 dB standard deviation. This calculation is repeated for a decrease of 0.13 in slope and for different SPL inputs, and the results averaged.

The standard deviation in slope may also be converted to SPL units by calculating the change in SPL at different input levels, created by a given change in the exponent of the Stevens equation (1955). This alternate calculation produces the same result.

The sone is the unit of loudness. The loudness, for normal listeners, of a sound of 40 dB SPL at 1 kHz (or 40 phons at any frequency) is assigned the value of 1 sone; the loudness in sones of a sound of any other physical amplitude is equal to the number of times the sound is louder or softer than 1 sone.
A loudness function plots the relation between the loudness and physical amplitude of sound, without providing information on whether successive loudness intervals have been judged arithmetically, geometrically, or some other way. However, if it is accepted that loudness-interval judgments are not made in a random fashion but on the basis of divisions that are essentially either arithmetic or geometric—a conclusion supported by the work of early psychoacousticians such as Stevens and Davis (1983) and by the results of the present experiments—a loudness function can be constructed from judgments of loudness intervals, as described below.

**Plotting a Loudness Function From Loudness-Interval Trisections**

To measure a loudness function from loudness-interval trisections the SPLs of the subject's responses are plotted against preset arithmetic or geometric dividers on the loudness axis (depending on the type of interval used by the subject), which makes loudness the parameter or independent variable of the data. The loudness/SPL coordinates used to construct a loudness function from arithmetic sone-interval trisections will be at different points of the curve than they would be for geometric sone-interval trisections, as illustrated in Figures 2a, b, but the shape of the curve will not be affected by the type of judgment used. The use of preset loudness coordinates is discussed further in the Discussion section.

The validity of the above method of measuring loudness functions is tested in experiment 1 by measuring the loudness functions of normal-hearing subjects by that method and comparing the results with the accepted normal loudness function. The relation between loudness and physical amplitude for an average normal listener is defined in ANSI Standard S.0.20–1973 (see footnote 4) by the equation:

\[ n_s = 2^{(L - 40)/10} \]

where \( n_s \) is loudness in sones and \( L \) is loudness level (see footnote 5) in phons. This equation is derived from the experimental determination that loudness for an average unimpaired listener is doubled for every 10-phon increase in sound pressure and halved for every 10-phon decrease (see footnote 6). The original equation proposed by Fletcher (1953) used 9 phons rather than 10 phons as the denominator of the exponent, on the basis of experimental data then available, but the 10-phon value for creating a twofold change in loudness is now recorded in ANSI Standard S.0.20–1995 and ISO Standard R131–1959E. ANSI Standard S.0.20–1973 (1973) states that this relation has been confirmed over the range of 20 to 120 phons.

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4ANSI S.0.1–1994 and ANSI S.0.20–1995 published a loudness-function equation that is inconsistent with the equation of the earlier Standard cited here. The equation of the later Standards was corrected in a 2006 corrigendum in *J Acoust Soc Am*, 118(2), 1290.

5Loudness level, in spite of its name, is a measure of the physical amplitude of a sound. The loudness level of a sound in phons, is numerically equal to the dB SPL of a 1-kHz sound of the same loudness. The SPL of a sound of given phon value varies with frequency according to the equal-loudness contours.

6The exponent of the equation represents the number of times the sound level has been increased or decreased by 10 phons from the reference level of 40 phons, and is thus the number of times the loudness has been doubled or halved from 1 sone. Two raised to this exponent is the number by which the reference loudness of 1 sone has been multiplied or divided, which is to say the number of sones.

Stevens (1955) developed an equivalent expression for loudness using absolute rather than phon units: Loudness in sones = kSP\(^{0.0}\) where SP is sound pressure in \( \mu \text{Bar} \) or \( \mu \text{Pa} \) units and \( k \) is a constant that ties the sound-pressure scale to the sone scale. \(^\text{For } \mu \text{Bar} \text{ units, } k = 10.54.\)

The Stevens loudness-function equation, like the ANSI equation used in this article, was derived from the experimental determination that for normal listeners a twofold change of loudness is created by a 10-dB change of sound pressure at 1 kHz (or a 10-phon change at any frequency).
EXPERIMENT 1: MEASUREMENT OF LOUDNESS FUNCTIONS BY TRISECTION

Subjects and Equipment

Nine normal-hearing subjects were used, seven of whom were inexperienced in listening to test signals. The range of age was 9 (subject MV) to 55, with 7 subjects between 20 and 30. The signal presentation was monaural.

Subjects were seated in a soundproof room and heard signals through TDH-39 earphones housed in an experimental circumaural mounting described by Villchur (1970). This mounting was designed to provide reduced intersubject and cushion-fit response variation compared to that of the standard MX41/AR supra-aural cushion mounting, and was calibrated by a real ear, free-field-reference method (Villchur, 1969).

Signals presented to the subjects were generated either by a General Radio audio generator 1304-B for sine waves, or a General Radio random-noise generator, a General Radio pink noise filter 190-P2, and a General Radio sound and vibration analyzer 1564 for narrowband noise. The presentation pattern of signals was controlled by a Grason-Stadler programmer 1201, with program control panel 1231 and Grason-Stadler electronic switches 829D. Signals were fed to the subjects by a Grason-Stadler audiometer 162 with fine adjustment by a Grason-Stadler recording attenuator E3262A. Subject responses were measured by a Ballantine vacuum-tube voltmeter 300H.

Subjects held a control box in their laps, and made adjustments with two large, unmarked potentiometer knobs. The potentiometers had logarithmic tapers and were part of constant-resistance circuits which provided smooth adjustments. One knob controlled the lower-trisection level and the other the higher-trisection level.

Method

Subjects were asked to trisect a series of loudness intervals, and their responses were used to plot their loudness functions. Each subject was presented with a series of four successive 1-kHz tones in ascending order of amplitude. The levels of the first and last tones were fixed at 40 and 80 dB SPL at the ear; subjects were instructed to adjust the levels of the second and third tones to create an even progression of loudness from the first to the last tone. Instructions that might have suggested that the subjects use either arithmetic or geometric intervals, such as “adjust for even distances of loudness between tones,” or “adjust for one-third of the loudness,” were avoided. Each tone had a duration of 0.5 sec, the interval between tones was 750 msec, and the tones were given a rise or decay time of 50 msec to avoid clicks. The series of tones was repeated until the subject signaled that he or she had completed the task, and the levels chosen for the second and third tones were recorded.

After the trisection of the 40 to 80 dB interval, subjects were asked to trisect two additional intervals: one between 40 dB SPL and the SPL the subject had chosen for the third tone of the 4-tone series, and one between the SPL the subject had chosen for the second tone of the series and 80 dB SPL.

As will be seen in the Results and Discussion sections, all subjects used close to arithmetically spaced loudness intervals, and so the SPLs of the subjects’ responses were plotted against fixed tone values on the loudness axis that represented ideal arithmetic intervals for each of the three trisections. The scale of the loudness axis was from 1 to 16 sones (40 dB SPL at 1 kHz is 1 sone; 80 dB is 16 sones). The loudness coordinates against which subject responses were plotted were at 4.33, 6, 7.66, 9.33, 11, and 12.66 sones. Because these predetermined sone

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TABLE 2. Test-retest measurements for three subjects.

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Values served as loudness coordinates for the SPL responses of the subjects, the loudness-function curve plotted the SPL corresponding to a given loudness rather than the loudness corresponding to a given SPL.

Preliminary measurements of this type were made on two subjects using 1-kHz tones in descending rather than ascending order of amplitude. Loudness functions were also measured at 0.5, 2, and 4 kHz, and with 1/3-octave pink noise centered at 1 kHz rather than sine wave signals. Test-retest reliability was measured by retesting three of the subjects on days subsequent to their original tests with three additional series of trials each.

Results

A comparison of trisection responses to signals presented in descending as opposed to ascending order of amplitude, together with corresponding values calculated from the loudness-function equation for both arithmetic and geometric loudness intervals, is shown in Table 1. Subject LM's responses to signals in descending order created some intervals that were neither arithmetic nor geometric, but leaned toward the arithmetic. Subject EV's descending-order trisections created geometric sone intervals, although not as close to ideal intervals as for the ascending-order presentations. Ascending-order presentations were used in all subsequent tests.

The loudness functions measured with sine wave signals at frequencies other than 1 kHz were essentially the same as those measured at 1 kHz. This result is consistent with the report of Stevens and Davis (1983) that normal loudness functions for tones between 700 Hz and 4 kHz are not significantly different from the loudness function for a 1-kHz tone. The loudness functions measured with 1/3-octave pink-noise signals were also essentially the same as those measured with 1-kHz sine wave signals.

The test-retest measurements of reliability are shown in Table 2. The average change of response from the first test to the second and third test, for the three subjects and six response levels (36 retest responses), ignoring sign, was 1.6 dB. The across-subject standard deviation of the test-retest differences was less than 1.7 dB for the first three lower-interval responses and less than 0.8 dB for the remaining responses.

Table 3 presents: (1) the six response SPLs of each of the 9 subjects for 1-kHz signal presentations

### Table 3. Six Trisection adjustments, in dB SPL, of each of nine subjects, and the standard deviation and average of their responses.

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The column labeled $n_c = 2^{-40/10}$ shows values of the normal loudness function predicted by that equation for these trisection levels. SPLs and standard deviations in bold type represent values for the first trisection; numbers in the last column identify responses for the first, second, and third trisection. All subject measurements were made to the nearest 1/4 dB, but averages and standard deviations were calculated to the nearest 1/10 dB.
between 40 and 80 dB SPL; (2) the standard deviation of the nine subject responses at each trisection level; (3) the average of the nine subject responses at each trisection level; and (4) SPLs predicted by the loudness-function equation for these trisection levels. The SPLs and standard deviations in bold type represent values for the first trisection; the numbers in the last column identify responses for the first, second, and third trisection.

For all six trisection levels, the average difference between individual subject responses (column labeled “Aver”) and values of the loudness-function equation (column labeled “$2^{(L-40)/10}$”) at the corresponding loudness coordinates was 0.78 dB ignoring sign.

At higher levels the variation in subject responses was reduced, and the responses came closer to the corresponding values of the loudness-function equation. The variation in test-retest responses shown in Table 2 also decreased at higher levels. This tightening of response variation is consistent with the known phenomenon that the higher the sound level the smaller the change in sound pressure required to produce a given change of loudness.

None of the subjects, including the 9-year-old, complained of the difficulty of their task.

Figure 3a compares the theoretical loudness function for normal listeners defined by the ANSI equation with the average trisection responses of the nine subjects, and shows the standard deviation of subject responses at each trisection level.

Figure 3b repeats the comparison in Figure 3a and adds a comparison between subject trisection responses and a loudness function derived from the Fletcher proposal that loudness doubles with a 9-dB increase of SPL. The maximum difference between average subject responses and a plot of the Stevens equation is 1.9 dB, compared to a maximum difference of 3.2 dB for the Fletcher equation. The measurement is sensitive enough to pick up the relatively small difference between the 10-dB and 9-dB loudness doubling.

**Discussion**

Subjects were free to choose any SPLs, and the close approximation of their responses to values calculated from an accepted equation for the normal loudness function is evidence of the validity of the trisection procedure for measuring a loudness function.

All subjects relied on arithmetic rather than geometric intervals for their responses, as indicated by the sone values of their choices of successive loudness intervals. Perfect arithmetic trisection of the loudness range between 1 and 16 sones would be at five-sone intervals, at 6 and 11 sones; the averages of the subjects’ first trisection choices between 40 and 80 dB were 64.8 and 74.3 dB, corresponding to 5.6 and 10.8 sones and creating loudness intervals of 4.6, 5.2, and 5.2 sones. Each of the loudness intervals judged to be equal had approximately the same

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**Fig. 3.** a. Solid line: Normal loudness function defined by the equation $n_s = 2^{(L-40)/10}$. Solid squares are the averages of the SPL choices of the 9 subjects at each trisection level; the line through each square represents the intersubject standard deviation of subject choices at that trisection level. The average standard deviation is 1.1 dB. b. Curve in heavy line is the normal loudness function as in Figure 3a, derived from the current Standard that a 10-phon change in amplitude creates a twofold change in loudness. Corresponding subject responses are shown by solid squares. Curve in light line is the loudness function derived from Fletcher’s hypothesis that a 9-phon change in amplitude creates a twofold change in loudness. The subject responses (X) plotted against the latter curve had to be adjusted to a vertical scale in which 80 dB corresponds to 21.77 rather than 16 sones.
number of sones, meaning that the subjects relied on arithmetic loudness intervals. Had the subjects trisected the range between 1 and 16 sones geometrically, they would have chosen SPLs that multiplied sone values by 2.52 (the cube root of 16/1) at each step, each signal judged 2.52 times as loud as the previous one. These SPL responses would then have been plotted against geometrically spaced loudness coordinates, but the shape of the resulting loudness-function curve would have been the same.

Although the loudness responses found in the present trisection study were arithmetic, preliminary results of a separate experiment indicate that musicians use geometric rather than arithmetic loudness intervals to produce the amplitude differences between the dynamic loudness markings found in music (pp, p, mp, mf, etc.).

It may seem counterintuitive to determine the fixed reference trisections of the loudness axis of the loudness-function graph only after the subjects’ responses have been taken into account. If the raw data of the subjects’ interval responses are to be incorporated into a curve relating sound pressure and loudness, some principle governing the subjects’ choices of intervals must be discovered. That principle was the subjects’ use of essentially arithmetic intervals, an experimental result that determined the fixed divisions of the loudness scale. If listeners divided amplitude ranges without following any particular mathematical principle, there would be no way to determine the present values of loudness against which subject SPL responses are plotted.

The subject’s trisection choices could have been plotted in another, equivalent way. Subject JC, for example, trisected the 40 to 80 dB SPL range at 64.75 and 74.25 dB. The ideal arithmetic loudness trisection for 5-sone intervals would be at 65.8 and 73.6 dB. Without making any assumptions it can be stated that subject JC’s first-interval choice was 1.05 dB below an ideal arithmetic loudness trisection, and her second-interval choice was 0.65 dB above it. Her loudness function could be plotted relative to the theoretical loudness function of Figure 3a from the differences between her trisection responses and ideal arithmetic trisections.

The consistency of subject responses was partly dependent on use of the experimental circumaural earphone mounting, whose combined intersubject and cushion-fit variation of response at 1 kHz was reduced to approximately half that of the MX41/AR supra-aural cushion. The experimental earphone never went into production, but reduced cushion-fit variation from that of the MX41/AR (not, however, at 1 kHz) is available today from insert audiometer earphones (Wilber et al., 1988).

**Experiment 2: Validation of the Trisection Procedure by Monaural Masking**

In experiment 2, the validity of the trisection procedure in measuring loudness functions is tested in a second way: masked-ear loudness functions measured by trisection are compared with masked-ear loudness functions measured by loudness-balance tests against the opposite unmasked ear.

Loudness-balance measurements between the normal and impaired ears of a unilaterally impaired subject have been used to establish loudness-function abnormalities by investigators such as Dix et al. (1948), Harris et al. (1952), and Steinberg and Gardner (1937). The results of these investigators were consistent with one another, and have good face validity.

**Subjects and Equipment**

The equipment was the same as that of experiment 1. Three of the subjects of experiment 1 were used. In our masking experiment, trisection SPLs were plotted against points of arithmetic divisions of the masked loudness range, not against sone values.

**Method**

The previously unmeasured ear of each subject was now masked by steady third-octave pink noise centered at 1 kHz, and trisection loudness-function measurements of the masked ears were made as in experiment 1. The masking level was adjusted to provide a convenient raising of the 40 to 80 dB range, shown in Figure 4. The SPL range of signals presented for trisection to the masked ear was established by loudness-balance tests against the limits for the unmasked measurements of experiment 1; the lower limit of the masked range for each subject was the masked signal whose SPL the subject adjusted to be equal in loudness to the loudness of an unmasked 40-dB SPL signal presented to the opposite ear, and the upper limit of the masked range was the masked signal whose SPL the subject adjusted to be equal in loudness to the loudness of an unmasked 80-dB SPL signal presented to the opposite ear. Signals of the same duration, separation time, and rise or decay as those used in experiment 1 were presented alternately to each ear until the subject signaled that his or her choice had been made.

The loudness function of the unmasked ear of each subject was measured by trisection with the other ear still masked, so that the possible effect of contralateral masking would be the same for both loudness-function and loudness-balance measurements. A second loudness function for the masked ear was then measured by loudness-balance measurements against signals to the unmasked ear: the
subject adjusted the amplitude of the masked signals to match the loudness of unmasked signals presented to the opposite ear at each of the six trisection SPLs of the unmasked responses.

The sone equation does not apply to SPL responses in the masked condition; under masking (or with recruitment) the 10-phon amplitude change that creates loudness doubling is reduced, so that the denominator of the sone-equation exponent becomes less than 10. The equation would not apply even if this factor were adjusted, because the recruitment curves reflected by the masked responses are not linear but asymptotic to the normal loudness function (see footnote 5) (plotted in dB on both axes as in Figure 5), indicating that the ratio between amplitude change and loudness doubling changes with level. Further, a signal heard under masking or by a hearing-impaired

<table>
<thead>
<tr>
<th>Subject MB</th>
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<th>Subject LM</th>
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trisection with the SPLs of a masked loudness function measured by loudness balance against the unmasked ear. The average difference in SPL between these two ways of measuring loudness functions for three subjects (18 responses for the trisections and 18 for the loudness-balance tests), ignoring sign, was 1.3 dB. Subject LM, who showed the greatest differences, made an unsolicited comment that she was uncertain about the loudness-balance choices because she heard different pitches in left- and right-ear signals, indicating that she had binaural diplacusis. This condition did not affect trisection judgments but compromised binaural loudness-balance judgments.

Figure 4 compares masked loudness functions measured by trisection with loudness functions measured by loudness balance between masked and unmasked ears, but unlike Table 4 in which all responses are included, Figure 4 includes only the responses of the two subjects who did not have binaural diplacusis.

Discussion

Loudness functions measured by loudness-balance measurements between masked and unmasked ears were consistent with loudness functions measured by the trisection procedure, providing additional evidence of the validity of the trisection procedure for measuring loudness functions.

Figure 4 shows the correspondence between trisection and loudness-balance measurements, but this correspondence by itself does not validate the trisection procedure. The loudness-balance measurements were made against SPLs of the unmasked trisection responses, and accurate loudness-balance measurements would retain any errors in the trisection measurements. However, Figure 4 also shows that if the loudness-balance measurements had been made against the theoretical loudness function, which is independent of the unmasked trisection responses, they would match the masked trisection responses as well as measurements made against the unmasked responses. The theoretical loudness function is a necessary part of the validation design.

If the trisection procedure is to be used with a hearing-impaired subject, the SPL limits of the range of signals presented at each frequency should have the same loudness for the impaired subject as the corresponding 40- and 80-dB SPL signals have for normal-hearing listeners. If the lower limit of the SPL range for an impaired subject were calculated by adding his or her hearing loss (HL) at the frequency being tested to the original 40-dB SPL used for normal listeners, that lower limit would be too high because recruitment—the accelerated increase of loudness as the SPL increases above threshold—would not be taken into account. The lower limit of the trisection presentation for a hearing-impaired subject can be calculated roughly if a fraction of the HL is added to 40-dB SPL, a fraction that gets larger as the hearing loss increases. For losses between 20 and 60 dB, HL/20 is added (at 20-dB hearing loss nothing is added); HL/20 is a progressively larger fraction of the HL as HL increases. Above losses of 60 dB, 0.5 HL + 10 is added, making the lower limit 50 + 0.5 HL.

The upper limit to the trisection presentation for a hearing-impaired subject can also be calculated roughly if an assumption is made of complete recruitment at 80-dB SPL for losses of 40 dB or less (which leaves the upper limit of 80 DB unchanged for such losses); 10 dB is added to the 80-dB limit for losses between 40 and 60 dB, and 20 dB is added for losses greater than 60 dB.

These estimates are derived from data in the instruction manual for Etymotic’s FIG6 fitting procedure (Killion, 1996). The method of setting test-range limits for hearing-impaired subjects may be modified by future experience, but even first approximations can make it possible to plot abnormal loudness functions accurately enough to guide the design of compensatory compression.

General Discussion

The procedure described here is capable of measuring the loudness functions of individual subjects with normal hearing or simulated hearing loss reliably and in detail, with significantly less measurement variation than has been reported for previous methods (except for binaural loudness balance, a procedure restricted to monaurally impaired subjects). As in loudness-balance measurements, subjects using the trisection procedure are asked to make acoustical comparisons rather than open-ended or cross-modality judgments, which would lead one to expect reduced variability.

Hearing aid compression ratios and the shape of the curve that plots compression ratio versus input level are calculated from loudness-function data presented in the form of the recruitment curves of Figures 1 and 5. The horizontal (or vertical) distances between the recruitment curve and the straight-line normal reference show the differences, over a range of input levels, between SPLs that produce a given loudness in a normal ear and SPLs that produce the same loudness in an impaired ear. In Figure 5 these differences may be thought of as representing subject MB’s simulated hearing loss (see footnote 7) as it varies over an input range of 40 to 80 dB. They also represent the level-dependent gain required of a compression amplifier if

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1 Harris et al. (1952) reported that while the recruitment curves associated with genuine hearing impairment varied in shape (as shown in Fig. 1), masking of a normal ear always created asymptotic recruitment curves.
the gain is designed to match the hearing loss at each input level. For example, MB’s masked ear requires 72.5 dB SPL input to hear the same loudness heard by a normal ear at 65 dB SPL. If a compression amplifier compensates the loss exactly at each level, 7.5 dB gain (72.5 dB to 65 dB) is required at 65 dB input.

The compression ratio for a given input range is the range of input (the vertical scale of Figure 5) divided by the smaller increase of output (the horizontal scale of Figure 5):

$$CR = \frac{\text{dB SPL}(1)_{in} - \text{dB SPL}(2)_{in}}{\text{dB SPL}(1)_{out} - \text{dB SPL}(2)_{out}}$$

where CR is the compression ratio, SPL(1)_{in} and SPL(2)_{in} are the limiting SPLs of a range of input signals, and SPL(1)_{out} and SPL(2)_{out} are the corresponding limiting SPLs of the range of amplified output signals.

Because the compression ratio is the input change in dB divided by the output change in dB, the compression ratio at a given level is numerically equal to the slope of the recruitment curve at that level. The recruitment curve for subject MB’s masked ear calls for a high compression ratio at low levels, tapering off to a value of 1.0 (no compression) as the level increases. A hearing aid compressor whose compression ratio does not vary with level would be adjusted to match a compromise straight-line recruitment curve that best approximates the subject responses.

It is possible that some hearing-impaired patients—particularly those for whom an effective hearing aid fitting has been elusive—need hearing aids whose compression/intensity characteristics can be adjusted to match the particular shapes of their loudness functions in different frequency regions. A person with the asymptotic recruitment described above might be one example; a person with the delayed recruitment of Figure 1 might be another. In the delayed-recruitment curve, recruitment starts at a higher level than it does for the other curves of Figure 1. A hearing aid needs to provide linear gain up to the onset of recruitment, a threshold of compression that matches the delayed recruitment, and a compression ratio that matches the recruitment slope above the threshold of compression (see footnote 8). The typical compression hearing aid cannot meet this need: it is designed to compensate for the “straight” type of recruitment curve shown in Figure 1 and has a fixed compression threshold.

The delayed-recruitment curve presents the greatest mismatch to the input/output curve of a typical hearing aid compression amplifier, which suggests that the fitting tactic of trying a higher threshold of compression, when possible, may be useful. The differences between the straight and asymptotic recruitment curves of Figure 1 do not appear to be great, but if the hearing threshold for the straight curve is moved up to match the threshold of the asymptotic curve at 40 dB, it becomes clear that the differences can be significant.

Whether it is useful to adjust the compression characteristics of a hearing aid to compensate for the shape of a particular patient’s loudness function needs to be determined experimentally. The amount and frequency of variation in the shape of recruitment curves must also be determined experimentally. Although Harris et al., did report measurements of the different shapes of recruitment curves among hearing-impaired subjects, they did not report on the frequency of occurrence of each shape. Measurements of the loudness functions of a large number of hearing-impaired subjects are needed to establish the amount of variation and frequency of occurrence of the different shapes of recruitment curves. It may be possible to simplify the trisection measurement by working out a procedure in which a larger number of divisions allows enough data to be collected in one trial.

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REFERENCES


